Equivalent Circuit Model for a Simple Slot-Shaped DGS Microstrip Line

Duk-Jae Woo, Taek-Kyung Lee, *Member, IEEE*, and Jae W. Lee, *Member, IEEE*

***Abstract*—In this letter, an analytic expression is derived for the resonance frequency of a simple slot-shaped DGS. In equivalent circuit modeling, the capacitance and inductance of the resonance circuit are evaluated from the dimensions of the DGS structure. The effects of equivalent circuit elements of a one-turn spiral DGS in the ground plane and their magnetic coupling to the host transmission line are modeled as a simple lumped-element circuit. The resonance frequencies calculated from the proposed method are compared with those obtained by EM simulation.**

***Index Terms*—Equivalent circuit model, magnetic coupling, resonance frequency, slot-shaped DGS.**

I. Introduction

A PLANAR transmission line with slot-shaped DGS and/or dumbbell-shaped DGS is one of the most interesting structures due to its unique characteristics including the band-gap effect and the slow-wave effect [1]–[3]. The DGS structures exhibit advantages such as easy fabrication and compatibility with monolithic microwave integrated circuits. Since the transmission line with DGS shows band rejection characteristics, it can be modeled simply by a parallel L-C resonant circuit and the parameters are extracted from EM simulated transmission characteristics. For dumbbell-shaped DGS, the equivalent circuit of a unit cell is composed by quasi-static modeling and the relationships between the physical dimensions of the defects and the parameters of the equivalent circuits are provided [4], [5].

The spiral DGS is a kind of slot-shaped DGS and it shows High-Q resonance characteristics compared to the dumbbell-shaped DGS [6], [7]. The slot-shaped DGS has a large group of variations from the spiral DGS according to the required performance. Many other researchers have presented equivalent circuits for various slot-shaped DGSs. A simple parallel RLC resonant circuit model was sufficient to provide electromagnetic behavior of slot-shaped DGS [8], [9]. Also, to explain the slot behavior over wide bandwidth, geometrical models based on transmission lines were proposed in [10]–[12]. These efforts have provided an improved physical insight of the operation principle of the DGS. In the design of a slot-shaped DGS, the most important property is the resonance frequency. To obtain the specified resonance frequency of the slot-shaped DGS, iterative calculation by an EM simulator is necessary; however, the process is very inefficient. To overcome this limitation, it is important to find an analytic expression for the resonance frequency, which can be directly derived from the physical structure of the slotted pattern.

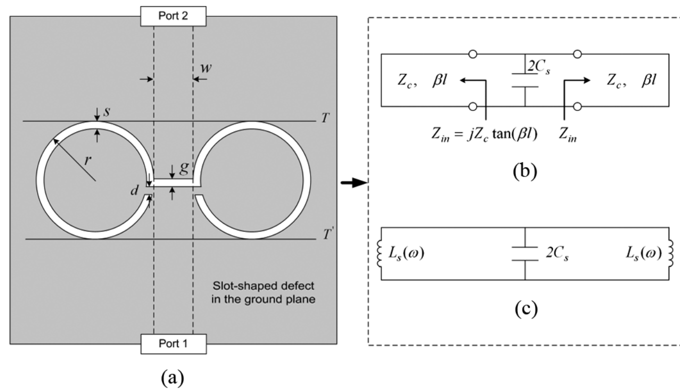

Fig. 1. (a) Configuration of microstrip line with a slot-shaped DGS. (b) Short ended slot-line transmission line model and (c) equivalent inductance model for the etched slot pattern in the ground plane.

In this letter, we propose a technique to obtain an analytic expression of the resonance frequency when the microstrip line is above a simple slot-shaped DGS structure. For this purpose, the equivalent circuit model for a one-turn spiral DGS coupled to a microstrip line is investigated. The proposed equivalent circuit model and analytic expression for the resonance frequency provide insight into the coupling mechanism of a DGS structure and design rules for a simple slot-shaped DGS structure.

II. Resonance Frequency

Fig. 1(a) illustrates the geometry of the microstrip line with a slot-shaped DGS where two etched circular slots are connected by a narrow etched gap. In the DGS, as the electric field is concentrated around the etched gap with a small dimension, the narrow etched gap can be modeled as a quasi-static capacitance, while the etched circular slots can be seen as two short-circuited slot-line TLs (transmission lines) with the characteristic impedance of Z_c and the length of l [10].

According to this, the DGS on the ground plane can be modeled as shown in Fig. 1(b), in which two slot-line TLs are short ended and are connected in parallel with the gap capacitance of $2C_\mathrm{s}$.

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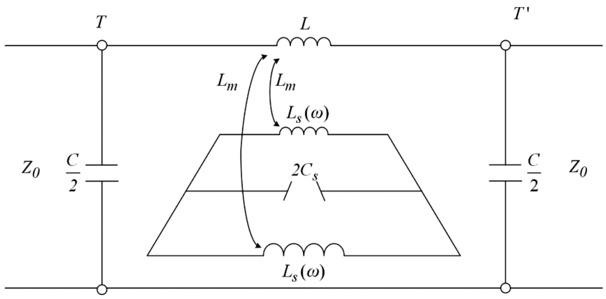

Fig. 2. Lumped element equivalent circuit model for a microstrip line with a slot-shaped DGS.

To operate the DGS as a resonant circuit, the input impedance $Z_{in}(= jX_{in})$ of the short-circuited slot-line TL of length l has to have an inductive reactance as

$$X_{in} = Z_c \tan(\beta l) = Z_c \tan\left(\frac{2\pi f\sqrt{\varepsilon_e}}{c}l\right) > 0 \qquad (1)$$

where the slot-line length l can be approximated by $2\pi(r - s/2) - d$, where c is the speed of light, ε_e and Z_c represent the effective permittivity and the characteristic impedance of the slot-line TL, respectively.

Then we can identify that the equivalent inductance $L_s(\omega)$ for the one-turn spiral DGS on the ground plane as depicted in the equivalent circuit of Fig. 1(c) is

$$L_s(\omega) = \frac{X_{in}}{\omega} = \frac{Z_c}{2\pi f}\tan\left(\frac{2\pi f\sqrt{\varepsilon_e}}{c}l\right). \qquad (2)$$

Finally, for the one-turn spiral DGS loaded microstrip line, the equivalent circuit model is shown in Fig. 2. The L and C are the inductance and capacitance of the microstrip line corresponding to the length occupied by the DGS, while the DGS on the ground plane is modeled as a resonant tank (with inductance $L_s(\omega)$ and capacitance C_s) magnetically coupled to the host line through a mutual inductance, L_m. The mutual inductance L_m is given by $L_m = k_m\sqrt{LL_s(\omega)}$, where k_m is the magnetic coupling coefficient between the host transmission line and the resonator.

Let us now focus on finding the resonance frequency f_0 of the equivalent circuit in Fig. 2. In Fig. 2, the resonance angular frequency is given by

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{L_s(\omega_0)C_s}}. \qquad (3)$$

Combining (2) and (3) for the resonance frequency, we can get

$$Z_c 2\pi f_0 \tan\left(\frac{2\pi f_0\sqrt{\varepsilon_e}}{c}l\right)C_s - 1 = 0. \qquad (4)$$

The closed form expressions for ε_e and Z_c were reported in [13], where the expressions have been obtained by curve fitting of the results based on Cohn's analysis. In the DGS on the ground plane, the narrow etched gap can be modeled as a microstrip gap as suggested by [4], and a closed form expression for microstrip gap capacitance $2C_s$ can be obtained from [11].

Having determined parameters, we can find the resonance frequency f_0 from (4). The resonance frequency was calculated with the help of MATLAB. Also, the equivalent inductance $L_s(\omega_0)$ at the resonance frequency can be calculated by using (2).

TABLE I
EXTRACTED CIRCUIT PARAMETERS AND RESONANCE FREQUENCIES FOR VARIOUS OUTER RADIUS OF THE ETCHED SLOT CIRCULAR PATTERN

r (mm)	C_s (pF)	$L_S(\omega_0)$ (nH)	f_0 (GHz)
2.0	0.0305	76.747	3.291
3.0	0.0305	169.37	2.215
4.0	0.0305	293.56	1.683
5.0	0.0305	447.85	1.362
6.0	0.0305	631.22	1.148
7.0	0.0305	842.84	0.993

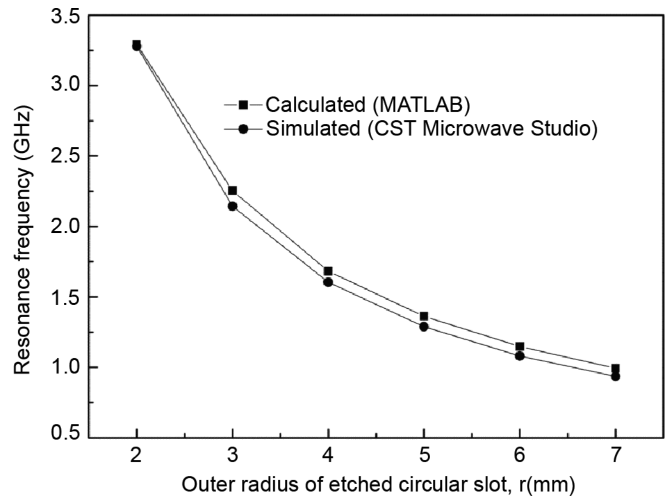

Fig. 3. Comparison of resonance frequencies of proposed model and EM simulation.

For the change in the outer radius of the etched slot, the resonance frequencies f_0 and the C_s and $L_s(\omega_0)$ at the resonance frequencies are calculated, which are summarized in Table I. In the design, a circuit board RO3010 with a dielectric constant of 10.2, copper thickness of 0.016 mm and substrate thickness of 1.27 mm is used. The characteristic impedance of the transmission line is designed to be $50\ \Omega(w = 1.2\ \text{mm})$, $g = s = 0.4\ \text{mm}$, and $d = 0.2\ \text{mm}$.

We compared the predicted resonance frequencies of the proposed model with the results from EM simulation. The simulation was done in the time domain solver of CST MICROWAVE STUDIO 2010. The resonance frequencies from the proposed model and those from EM simulation are plotted as functions of the outer radius of the etched circular slot in Fig. 3. The comparison shows that the proposed circuit model predicts the resonance frequencies with reasonable accuracy for a change in radius from 2 to 7 mm.

III. Simulation and Experimental Results

Near the resonant frequency, we can obtain the simplified equivalent circuit model, as shown in Fig. 4 [14], where

$$L_s' = 2\omega_0^2 L_m^2 C_s = 2k_m^2 L \qquad (5)$$

$$C_s' = \frac{L_s(\omega_0)}{(2\omega_0^2 L_m^2)} = \frac{L_s(\omega_0)C_s}{L_s'} \qquad (6)$$

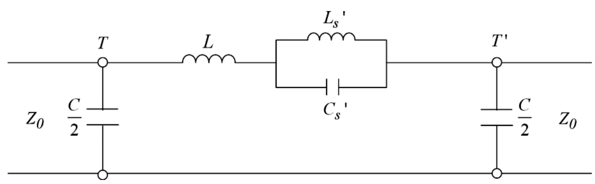

Fig. 4. Simplified equivalent circuit model.

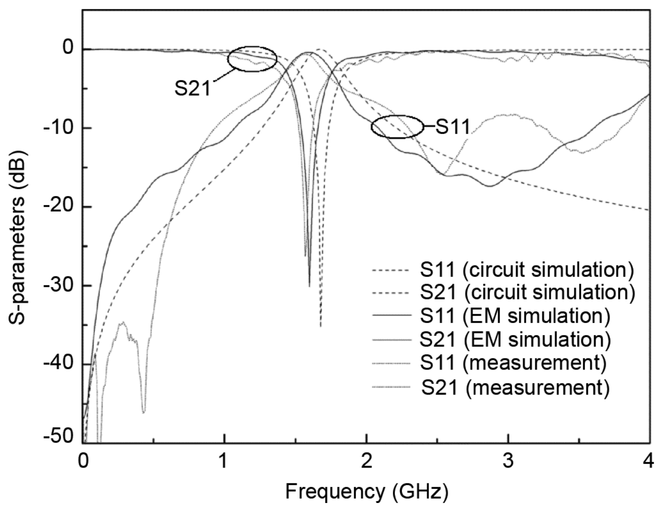

Fig. 5. S-parameters by circuit simulation, EM simulation, and measurement.

$$\omega_0 = \frac{1}{(2\pi f_0)} = \frac{1}{\sqrt{L_s' C_s'}} = \frac{1}{\sqrt{L_s(\omega_0) C_s}} \tag{7}$$

and the analytic expressions for L and C can be obtained from [15].

Having determined circuit parameters in the previous section with L and C, circuit simulation for the simplified equivalent model is performed by using the ADS circuit simulator. Fig. 5 illustrates the comparative S-parameters from the circuit simulation (ADS), EM simulation (CST), and the measurement for a fabricated DGS with $\mathrm{r} = 4\ \mathrm{mm}$. Other dimensions of the structure are the same with those used in the calculation of Fig. 3 in the previous section. The calculated circuit parameters are $L = 3.4163\ \mathrm{nH}$, $C = 1.4153\ \mathrm{pF}$, $L_s^{'} = 1.7772\ \mathrm{nH}$, and $C_s^{'} = 5.0336\ \mathrm{pF}$. The magnetic coupling coefficient k_{m} is chosen here to be 0.51. This optimum value is inferred from the 3 dB bandwidth of the EM simulation result.

As shown in Fig. 5, the resonance frequencies obtained from the circuit simulation, the EM simulation, and the measurement are 1.68, 1.6, and 1.58 GHz, respectively. It can be seen that the proposed method of circuit simulation provides a good approximation for the characteristics of a simple slot-shape DGS structure. The discrepancies may be attributed to the equivalent circuit model where the dimensions are very frequency sensitive and to the inductive reactance that could be involved at the end of short-circuited slot-line.

IV. Conclusion

This letter has presented an analytical expression for resonance frequency and an equivalent circuit model of a simple slot-shaped DGS. Reasonable agreement regarding the resonance frequencies between the proposed method and EM simulation has been obtained. The proposed equivalent circuit model provides an improved physical insight into the electromagnetic behavior of the slot-shaped DGS.

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